



BOOK OF ABSTRACTS  
6TH INTERNATIONAL CONFERENCE  
ON MATRIX ANALYSIS AND APPLICATIONS  
JUNE 15-18, 2017  
Duy Tan University, Danang, Vietnam



International Linear Algebra Society

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# General Information

## Proceedings

Acta Mathematica Vietnamica intends to publish an issue on Matrix Analysis and Applications and invites submissions by the participants of the 6th International Conference on Matrix Analysis and Applications (ICMAA 2017) at Duy Tan University, 15-18 June 2017. Papers should be submitted by December 15, 2017 via the journal system: <https://link.springer.com/journal/40306>. All submission will be properly reviewed according to the international standards.

## Sponsors

We would like to thank the following organizations for their direct or indirect support of the 6th International Conference on Matrix Analysis and Applications (ICMAA 2017):

- Duy Tan University, Vietnam
- International Linear Algebra Society (ILAS) sponsors the conference

## Instructions and suggestions for speakers

- All contributed talks are 20 minutes plus 5 minutes for questions.
- There is a projector in each room for your presentation.
- Laptops can be connected through VGA to the projector.
- It is always a good idea to load your presentation in PDF format on a USB stick.

## Excursion

Excursion will be arranged on Sunday, June 18, 2017.

Other information about attractions in Danang City can be found at <http://www.vietnam-guide.com/danang/attractions/>.

## Conference dinner

Banquet will be held on Friday evening June 16, 2017.

## Internet connection

Free internet connection is available in campus.

# Schedule of talks

6/16 (Friday)			6/17 (Saturday)		
8:00-8:40, Registration			8:10-8:55, A. Berman, Chair Q. Wang		
8:40-9:00	Opening Remarks University Officer T.Y. Tam, T.H. Dinh, F. Zhang		Time/Chair	R. Pereira@	Q. Wang@
9:00-9:15	Group Photo		8:55-9:20	M.T. Chien	R. Tabata
9:15-10:10	Keynote speaker M.D. Choi Chair T.Y. Tam		9:20-9:45	H. Nakazato	T. Jiang
			9:45-10:10	P.R. Huang	T.H. Le
10:10-10:30 Coffee/Tea break			10:10-10:30 Coffee/Tea break		
Time/Chair	J. Hou@	S. Kye@	Time/Chair	R. Pereira@	Q. Wang@
10:30-10:55	H. Du	S. Furuichi	10:30-10:55	S. Jayaraman	T.Y. Tam
10:55-11:20	H.K. Le	H. Ohno	10:55-11:20	P.S. Lau	P. Ha
11:20-11:45	J. Tao	M. Nguyen	11:20-11:45	X. Qi	Z.Q. Wang
11:45-12:10	J. Hou	S. Kye	11:45-12:10	P.D. Hoang	Q. Wang
12:10-1:30 Lunch			12:10-1:30 Lunch		
1:30-2:15, T. Ando, Chair H. Osaka			1:30-2:15, R. Brualdi, Chair T.H. Dinh		
Time/Chair	D. Chu@	T.H. Dinh@	Time/Chair	X. Zhang@	F. Zhang@
2:15-2:40	T.S. Pham	M. Cerny	2:15-2:40	G.L. Chen	X. Kong
2:40-3:05	M. Marciniak	K. Vo	2:40-3:05	H. Monterde	A.V. Le
3:05-3:30	T. Teh	M. Hladik	3:05-3:30	B.T. Du	K. Rodtes
3:30-3:45 Coffee/Tea break			3:30-3:45 Coffee/Tea break		
Time/Chair	D. Chu@	T.H. Dinh @	Time/Chair	X. Zhang@	F. Zhang@
3:45-4:10	H. Li	R. Pereira	3:45-4:10	K. Toyonaga	C.T. Le
4:10-4:35	Q.H. Phan	C.Arumugasamy	4:10-4:35	V.H. Ha	L. Liu
4:35-5:00	D. Chu	T.H. Dinh	4:35-5:00	X. Zhang	F.Zhang

Note: 1 Keynote (50+5 min) + 3 invited (40+5min) + 46 contributed (20+5 min) = 50 talks.

# Talk Titles and Abstracts

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**Title:** A Mathematician's Apology on Tensor Products

**Keynote Speaker:** Man-Duen Choi (choi@math.toronto.edu), University of Toronto, Canada.

**Abstract:** This is an expository lecture on the structure of tensor products of complex matrices. In all times of my mathematical journey, I have beautiful dreams of non-commutative geometry. Suddenly, I was awoken in the new era of Quantized world with fantasies and controversies. To release myself from Quantum Entanglements and the Principle of Locality, I need to seek the meaning of physics and the value of metaphysics. Conclusion: I THINK, THEREFORE I AM a pure mathematician.

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**Title:** Norm-ratios under a weak order relation in  $\mathbb{M}_m \otimes \mathbb{M}_n$

**Invited speaker:** Tsuyoshi Ando (ando@es.hokudai.ac.jp), Hokkaido University (Emeritus), Japan

**Abstract:** In the real Hilbert space of self-adjoint elements of the tensor product  $\mathbb{M}_m \otimes \mathbb{M}_n$ , there appear two natural cones besides the cone  $\mathfrak{P}_0$  of positive semi-definite elements. The one is

$$\mathfrak{P}_+ := \text{the convex hull of } \{X \otimes Y \mid 0 \leq X \in \mathbb{M}_m, 0 \leq Y \in \mathbb{M}_n\}$$

and the other is the cone  $\mathfrak{P}_-$ , dual to  $\mathfrak{P}_+$  with respect to the inner product. Then  $\mathfrak{P}_+ \subset \mathfrak{P}_0 \subset \mathfrak{P}_-$ . A weak order relation is introduced by

$$\mathbf{S} \succ \mathbf{T} \stackrel{\text{def}}{\iff} \mathbf{S} - \mathbf{T} \in \mathfrak{P}_-$$

We discuss ratio  $\|\mathbf{T}\|/\|\mathbf{S}\|$  for  $\mathbf{S} \succ \mathbf{T} \succ 0$  where  $\|\cdot\|$  is one of the operator norm, the trace-norm and the Hilbert-Schmidt norm. The dual forms of the results are also discussed.

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**Title:** Completely positive matrices – real, rational and integral

**Invited speaker:** Abraham Berman (berman@technion.ac.il), Professor Emeritus of Mathematics, Technion - Israel Institute of Technology, Israel

**Abstract:** A completely positive factorization of a matrix  $A$  is  $A = BB^T$ , where  $B$  is a nonnegative matrix. A rational completely positive factorization of a matrix  $A$  is  $A = BB^T$ , where  $B$  is a nonnegative matrix such that its entries are rational.

An integral completely positive factorization of a matrix  $A$  is  $A = BB^T$ , where  $B$  is a nonnegative matrix such that its entries are integers.

A matrix is completely positive if it has a completely positive factorization.

In the talk we will discuss the following questions:

1. When is a symmetric nonnegative matrix, completely positive?
  2. Does every rational completely positive matrix have a rational completely positive factorization?
  3. Does every integral completely positive matrix have an integral completely positive factorization?
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**Title:** Permutation Matrices, Alternating Sign Matrices, and the Bruhat Order

**Invited speaker:** Richard A. Brualdi (brualdi@math.wisc.edu), University of Wisconsin - Madison, USA

**Abstract:** Permutations Matrices are fundamental combinatorial and matrix objects. Alternating Sign Matrices (ASMs) are generalizations of permutation matrices. The Bruhat order is a partial order on the permutation matrices that extends to ASMs. There is also a matrix Bruhat decomposition. The Bruhat order relates to flags in vector spaces. And there is a determinant-like “alternating sign matrix polynomial” of the entries of a matrix. We shall discuss all these things.

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**Title:** Generalized Lyapunov transformation and semidefinite linear complementarity problems.

**Speaker:** Chandrashekar Arumugasamy (chandrashekar@cutn.ac.in), Central University of Tamil Nadu, India.

**Co-authors:** Gokulraj S (gokul15@students.cutn.ac.in), Central University of Tamil Nadu, India, and Sachindranath Jayaraman (sachindranathj@iisertvm.ac.in), IISER Thiruvananthapuram, India.

**Abstract:** Let  $S^n$  be the space of all real symmetric matrices of order  $n$  and  $S_+^n$  be the cone of all positive semidefinite matrices in  $S^n$ . Given a linear transformation  $L : S^n \rightarrow S^n$  and  $Q \in S^n$ . Then the semidefinite linear complementarity problem  $SDLCP(L, Q)$  is to find an  $X \in S_+^n$  such that  $L(X) + Q \in S_+^n$  such that  $\langle X, L(X) + Q \rangle = 0$ . For  $A, B \in M_n(\mathbb{R})$  with  $A$  or  $B$  orthogonal, we consider the generalized Lyapunov transformation  $L_{A,B}$  defined as  $AXB + B^T X A^T$  for all  $X \in S^n$ . We study the global solvability and uniqueness of solutions for the problem  $SDLCP(L_{A,B}, Q)$  similar to those of  $SDLCP$ 's with the Lyapunov transformation  $L_A(X) = AX + XA^T$ .

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**Title:** A Branch-and-Bound scheme for the range of a rank-deficient quadratic form with interval-valued variables

**Speaker:** Michal Černý (cernym@vse.cz), University of Economics, Prague, Czech Republic.

**Co-authors:** Miroslav Rada and Milan Hladík ({miroslav.rada, milan.hladik}@vse.cz), University of Economics, Prague, Czech Republic.

**Abstract:** Given a positive semidefinite quadratic form  $f(x) = x^T Q x$  and bounds  $\underline{x} \leq x \leq \bar{x}$  for the variables, we address the problem of computing the range  $\underline{f} = \min_{\underline{x} \leq x \leq \bar{x}} f(x)$  and  $\bar{f} = \max_{\underline{x} \leq x \leq \bar{x}} f(x)$ . The lower bound  $\underline{f}$  can be computed efficiently via CQP, while computation of the upper bound  $\bar{f}$  is NP-hard. We focus on the case when  $Q$  is rank-deficient. We reformulate the computation of  $\bar{f}$  as a problem of enumeration of vertices of a zonotope in  $d$ -dimensional space, where  $d = \text{rank}(Q)$ . Instead of constructing the enumeration in full, we run a B&B scheme. The branching step consists in a split of a zonotope into a pair of “smaller” zonotopes by removal of a generator. In the bound-part, we use Goffin’s method to approximate a zonotope by a pair of Löwner-John ellipsoids. Then, the lower and upper bound for  $f$  over an ellipsoid is computed by Ye’s algorithm for optimization of (arbitrary) quadratic forms over ellipsoids. We also discuss the impact of various strategies for the choice of (i) the active zonotope, (ii) the branching generator and (iii) the method for computation of lower bounds. (Supported by CSF 16-00408S.)

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**Title:** Generalized and generic inverse eigenvalue problems for pseudo-Jacobi matrices whose graphs are prescribed mixed paths

**Speaker:** Guo-Liang Chen (glchen@math.ecnu.edu.cn), East China Normal University, Shanghai, P.R. China.

**Co-authors:** Wei-Ru Xu (weiruxu@foxmail.com), East China Normal University, Shanghai, P.R. China.

**Abstract:** In this talk, we extend Jacobi matrix to two kinds of pseudo-Jacobi matrices  $J(H_1, n, r, \beta)$  and  $J(H_2, n, r, \beta)$  with mixed paths as their graphs in the non-self-adjoint setting, where  $H_1 = I_r \oplus -I_{n-r}$  ( $0 < r < n$ ) and  $H_2 = I_r \oplus -I_1 \oplus I_{n-r-1}$  ( $0 < r < n - 1$ ) are fixed in an indefinite inner product spaces and  $\beta = (\beta_1, \beta_2, \dots, \beta_{n-1})$  is positive. Some properties of eigenvalues and eigenvectors of the prescribed pseudo-Jacobi matrices are studied. Then we respectively investigate generalized inverse eigenvalue problem for the matrix  $J(H_1, n, r, \beta)$  and generic inverse eigenvalue problem for the matrix  $J(H_2, n, r, \beta)$ . Moreover, necessary and sufficient conditions are derived such that the problems are solvable. Finally, two algorithms to recover the proposed pseudo-Jacobi matrices from the known spectral information are presented and some illustrated numerical experiments are executed.

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**Title:** Construction of the determinantal representation of ternary forms

**Speaker:** Mao-Ting Chien (mtchien@scu.edu.tw), Soochow University, Taipei, Taiwan.

**Co-author:** Hiroshi Nakazato, Hirosaki University, Japan.

**Abstract:** Let  $A$  be an  $n$ -by- $n$  matrix. The ternary form associated to  $A$  is defined by  $F_A(x, y, z) = \det(x\Re(A) + y\Im(A) + zI_n)$ , where  $\Re(A) = (A + A^*)/2$  and  $\Im(A) = (A - A^*)/(2i)$ . Kippenhahn (1951) characterized that the numerical range of  $A$  is the convex hull of the real affine part of the dual curve of the curve  $F_A(x, y, z) = 0$ . The Fiedler-Lax conjecture is recently confirmed by Helton and Vinnikov, namely, for any real hyperbolic ternary form  $F(x, y, z)$ , there exist real symmetric matrices  $H$  and  $K$  such that  $F(x, y, z) = F_{H+iK}(x, y, z)$ . We explicitly construct real symmetric matrices  $H$  and  $K$  for the determinantal representation of some ternary forms and algebraic curves.

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**Title:** Least Squares Approach for Regularized Incremental Linear Discriminant Analysis on Large-Scale Data

**Speaker:** Delin Chu (matchudl@nus.edu.sg), University of Singapore, Singapore.

**Abstract:** Over the past a few decades, much attention has been drawn to large-scale incremental data analysis, where researchers are faced with huge amount of high-dimensional data acquired incrementally. In such a case, conventional algorithms that compute the result from scratch whenever a new sample comes are highly inefficient. To conquer this problem, we propose a new incremental algorithm IRLS that incrementally computes the solution to the regularized least squares (RLS) problem with multiple columns on the right-hand side. More specifically, for a RLS problem with  $c$  ( $c \geq 1$ ) columns on the right-hand side, we update its unique solution by solving a RLS problem with single column on the right-hand side whenever a new sample arrives, instead of solving a RLS problem with  $c$  columns on the right-hand side from scratch. As a direct application of IRLS, we consider the supervised dimensionality reduction of large-scale data and focus on linear discriminant analysis (LDA). We first propose a new batch LDA model that is closely related to RLS problem, and then apply IRLS to develop a new incremental LDA algorithm. Experimental results on real-world datasets demonstrate the effectiveness and efficiency of our algorithms.

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**Title:** Some new results on matrix means

**Speaker:** Trung-Hoa Dinh (trunghoa.math@gmail.com), University of North Florida, USA.

**Abstract:** In this talk we discuss in-betweenness and in-circleness properties for positive semidefinite matrices. We also provide an answer to a conjecture on the matrix power means due to Bhatia, Lim and Yamazaki.

**Co-authors:** Jose Franco and Raluca Dumitru from the University of North Florida.

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**Title:** Handelman's positivstellensatz for polynomial matrices which are positive definite on polyhedra

**Speaker:** Binh TH Du (hoabinhdsp@gmail.com), Ha Tay College of Pedagogy, Vietnam

**Abstract:** In this paper we give a matrix version of Handelman's Positivstellensatz [see Handelman, D.: Representing polynomials by positive linear functions on compact convex polyhedra. Pacific J. Math. **132**, 35-62 (1988)], representing polynomial matrices which are positive definite on convex, compact polyhedra. Moreover, we propose also a procedure to find such a representation. As a corollary of Handelmans theorem, we give a special case of Schmudgens Positivstellensatz for polynomial matrices positive definite on convex, compact polyhedra.

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**Title:** Block-operator technique and spectral theory approach in Krein space

**Speaker:** Hongke Du (hkdu@snnu.edu.cn), Shaanxi Normal University, Xi'an, P.R. China.

**Abstract:** In this talking, using block-operator technique and operator spectrum theory, we study the spectrum and geometry of regular subspaces of Krein space.

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**Title:** On estimates for Tsallis relative operator entropy

**Speaker:** Shigeru Furuichi (furuichi@chs.nihon-u.ac.jp), Nihon University, Tokyo, Japan

**Abstract:** After the review of previous results on (Tsallis) relative operator entropy, we give our recent results, the tight bounds of Tsallis relative operator entropy by the use of Hermite-Hadamard's inequality. We also give alternative tight bounds for the Tsallis relative operator entropy.

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**Title:** Analysis of delay differential-algebraic equations

**Speaker:** Phi Ha (phisp1802@gmail.com), VNU University of Science, Hanoi, Vietnam.

**Co-author:** Volker Mehrmann, Technical University of Berlin, Berlin, Germany.

**Abstract:** During the last 20 years, much research has been focused on differential-algebraic equations (DAEs). These systems appear in a wide variety of scientific and engineering applications, including circuit analysis, computer-aided design and real-time simulation of mechanical (multibody) system, power systems, chemical process simulation. On the other hand, much work has also been done in the field of delay differential equations (DDEs). Delay differential equations arise from, for example, real time simulation, where time delays can be introduced by the computer time needed to process the input data. Delays also arise in circuit simulation and power systems, due to, for example, interconnects for computer chips and transmission lines, and in chemical process simulation when modeling pipe flows.

Even though the theory of the analytical and numerical solution of delay differential equations (DDEs) as well as differential-algebraic equations (DAEs) is well understood, the intersection of them, the delay differential-algebraic equations (DDAEs), is still an open object, even for the relatively simple case of linear systems with constant coefficients.

In this talk, we discuss linear constant coefficient DDAEs. We first give an introduction about DDAEs and how they are different from both differential-algebraic equations as well as delay differential equations. We then study the solvability/stability analysis based on investigating the structure of the matrix coefficients.

**Reference:**

1. Phi Ha, Volker Mehrmann. *Analysis and numerical solution of linear delay differential-algebraic equations*. BIT Numerical Mathematics. (2016), 633657.
  2. Phi Ha. *Analysis and numerical solutions of Delay Differential-Algebraic Equations*. (2015) PhD Thesis, Technical University Berlin, Germany.
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**Title:** On the Maximal rank of Completions of Entry Pattern Matrices

**Speaker:** Van-Hieu Ha (hieu.havan@nuigalway.ie), School of Maths, National University of Ireland, Galway, Ireland.

**Co-authors:** Rachel Quinlan (rachel.quinlan@nuigalway.ie), School of Maths, National University of Ireland, Galway, Ireland.

**Abstract:** Given a set  $S = \{x_1, x_2, \dots, x_k\}$ , we denote by  $M_n(S)$ , or  $M_n(x_1, x_2, \dots, x_k)$  the set of  $n \times n$  matrices whose entries are from  $S$ . The following definition appears in the paper "Nonsymmetric normal entry patterns with the maximum number of distinct indeterminates" (authors: Zejun Huang, Xingzhi Zhan).

**Definition 1:** Let  $x_1, x_2, \dots, x_k$  be distinct indeterminates. We call a matrix in  $M_n(x_1, x_2, \dots, x_k)$  an *entry pattern matrix*.

Thus an entry pattern is a matrix whose entries are indeterminates, some of which may be equal. The *pattern class* of an entry pattern matrix  $A$ , denoted by  $C_F(A)$ , is the set of the matrices obtained by specifying the values of the indeterminates of  $A$  by elements in some field  $F$ . Each element in  $C_F(A)$  is called an  $F$ -completion of  $A$ .

**Definition 2:** The *maximum  $F$ -rank* of  $A$  is the maximum rank of  $F$ -completions of  $A$ . The *generic rank* of  $A$  is the rank of  $A$  when considered as a matrix in  $M_n(\bar{F})$ , where  $\bar{F}$  is the field of rational functions in the indeterminates that appear in  $A$  over  $F$ .

The maximum  $F$ -rank cannot exceed the generic rank, and these are equal if the field  $F$  is big enough or the number of indeterminates is smaller than 3. This talk will present some precise conditions under which the maximum  $F$ -rank and the generic rank coincide, and show that any finite field  $F_q$  of characteristic less than 17 (except  $F_2$  - the finite field of 2 elements) are EPM-rank-tight, i.e. there is a square EPM of order  $q + 1$  so that its  $F_q$ -generic rank can not attain by any completion on  $F_q$ .

**Title** Generalized regularity and singularity of interval matrices

**Speaker** Milan Hladík (hladik@kam.mff.cuni.cz), Charles University, Prague, Czech Republic.

**Abstract:** An interval matrix is called regular if it contains nonsingular matrices only. Herein, we introduce a novel concept of generalized regularity given by  $\forall\exists$ -quantification: An interval matrix is AE regular if for every realization of  $\forall$ -coefficients there is a realization of  $\exists$ -coefficients such that the resulting matrix is nonsingular.

The proposed concept strongly relates with linear systems of equations with interval coefficients. A vector  $x$  is an AE solution of such a if for every realization of  $\forall$ -coefficients there is a realization of  $\exists$ -coefficients such that  $x$  solves the corresponding system. We will see that AE regularity implies that the AE solution set is nonempty. We will also study characterization of AE regularity and discuss various classes of matrices that are implicitly AE regular. Some of these classes are polynomially decidable, and therefore give an efficient way for checking AE regularity.

**Title:** Lojasiewicz exponents of analytic function germs of two variables

**Speaker:** Phi-Dũng Hoang (dunghp@ptit.edu.vn), Posts and Telecommunications Institute of Technology (PTIT), Hanoi, Vietnam.

**Co-authors:** Hong-Duc Nguyen and Tien-Son Pham

**Lojasiewicz gradient inequality.** Let  $f : (\mathbb{K}^n, 0) \rightarrow (\mathbb{K}, 0)$  (where  $\mathbb{K} = \mathbb{C}$  or  $\mathbb{R}$ ) be an analytic function germ and  $\nabla f(0) = 0$ . The Lojasiewicz gradient inequality ([1, 2]) asserts that: There exist  $C > 0, \alpha \in [0, 1)$  and a neighbourhood  $U$  of 0 such that

$$\|\nabla f(x)\| \geq C|f(x)|^\alpha \quad \text{for } x \in U. \quad (1)$$

The number  $\alpha_{\mathbb{K}}(f) = \inf\{\alpha \mid \alpha \text{ satisfies (1)}\}$  is *Lojasiewicz exponent*.

**Classical Lojasiewicz inequality:** Let  $f : (\mathbb{K}^n, 0) \rightarrow (\mathbb{K}, 0)$  be an analytic function germ. Let  $V := \{x \in \mathbb{K}^n \mid f(x) = 0\} = f^{-1}(0)$  and  $K$  be a compact subset in  $\mathbb{K}^n$ . Then the (classical) Lojasiewicz inequality ([1, 2]) asserts that: There exist  $c > 0$  and  $\beta > 0$  such that

$$|f(x)| \geq cd(x, V)^\beta \quad \text{for } x \in K. \quad (2)$$

The number  $\beta_{\mathbb{K}}(f) = \inf\{\beta \mid \beta \text{ satisfies (2)}\}$  is the *Lojasiewicz exponent*.

If  $\mathbb{K} = \mathbb{R}$ ,  $f^{-1}(0) = \{(0, \dots, 0)\}$  then (2):  $|f(x)| \geq c\|x\|^\beta, \forall x \in K$ .

**Problem.**

*Research:*  $\alpha_{\mathbb{K}}(f)$  and  $\beta_{\mathbb{K}}(f)$ .

In this talk, we consider  $f$  be an analytic function germ of two (real or complex) variables. We show that the Lojasiewicz exponent in the gradient inequality (two variables) is attained along the polar curve of  $f$ . We used the method which is called sliding in [7], it was deeply developed by Vui-Duc in [6]. Moreover, we attained some effective computations of Lojasiewicz exponents.

**Effective estimates for the Lojasiewicz exponents.**

Corollary: Let  $f : \mathbb{K}^2 \rightarrow \mathbb{K}$  be a polynomial of degree  $d$ . Then

$$\alpha_{\mathbb{K}}(f) \leq 1 - \frac{1}{d(d-1)}.$$

This estimate is sharper than the result of D'Acunतो and Kurdyka (Ann. Polon. - 2005) ([4]).

**Corollary:** Let  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  be a real polynomial of degree  $d$ . We have

$$\beta_{\mathbb{R}}(f) \leq \frac{1}{1 - \alpha_{\mathbb{R}}(f)}.$$

In particular,

$$\beta_{\mathbb{R}}(f) \leq d(d - 1).$$

This estimate is sharper than the result of Kurdyka - Spodzieja (Proc. AMS - 2014) ([8]).

The Lojasiewicz inequalities have many applications in singularity theory ([11]), geometry, optimization ([5]) and eigenvalues optimization (for example [9]).

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- [2] S. Lojasiewicz, *Ensembles semi-analytiques*, Publ. Math. I.H.E.S., Bures-sur-Yvette, France (1964).
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- [5] J. Bolte, A. Daniilidis, O. Ley, L. Mazet, *Characterizations of Lojasiewicz Inequalities and Applications*, Transactions of the American Mathematical Society, vol. 362, n. 6, (2010), pp. 3319-3363.
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**Title:** Strong 2-Skew Commutativity Preserving Maps] Strong 2-Skew Commutativity Preserving Maps On Prime Rings With Involution

**Speaker:** Jinchuan Hou (jinchuanhou@aliyun.com), Taiyuan University of Technology, China

**Abstract:** Let  $\mathcal{R}$  be a unital prime  $*$ -ring containing a nontrivial symmetric idempotent. For  $A, B \in \mathcal{R}$ , the skew commutator and 2-skew commutator are defined respectively by  $_*[A, B] = AB - BA^*$  and  $_*[A, B]_2 = _*[A, _*[A, B]]$ . Let  $\Phi: \mathcal{R} \rightarrow \mathcal{R}$  be a surjective map. We show that (1)  $\Phi$  satisfies  $_*[\Phi(A), \Phi(B)] = _*[A, B]$  for all  $A, B \in \mathcal{R}$  if and only if there exists  $\lambda \in \{-1, 1\}$  such that  $\Phi(A) = \lambda A$  for all  $A \in \mathcal{R}$ ; (2)  $\Phi$  satisfies  $_*[\Phi(A), \Phi(B)]_2 = _*[A, B]_2$  for all  $A, B \in \mathcal{R}$  if and only if there exists  $\lambda \in \mathcal{C}_S$  with  $\lambda^3 = I$  such that  $\Phi(A) = \lambda A$  for all  $A \in \mathcal{R}$ , where  $I$  is the unit of  $\mathcal{R}$  and  $\mathcal{C}_S$  is the symmetric extend centroid of  $\mathcal{R}$ . This is then applied to prime  $C^*$ -algebras, factor von Neumann algebras and indefinite self-adjoint standard operator algebras to

get a complete invariant for the identity map, and to symmetric standard operator algebras as well as matrix algebras.

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**Title:** Product of two diagonal entries of a 3-by-3 normal matrix

**Speaker:** Peng-Ruei Huang (h16ds202@hirosaki-u.ac.jp), Hirosaki University, Hirosaki, japan.

**Co-authors:** Hiroshi Nakazato, Hirosaki University

**Abstract:** We will consider the shape of the following range

$$W_k^\Pi(N) = \left\{ \prod_{i=1}^k (UAU^*)_{ii} : U \text{ is a unitary matrix} \right\}$$

when  $k = 2$  and  $N$  is a  $3 \times 3$  normal matrix. The convexity of the above range for a  $3 \times 3$  normal matrix will be concerned. The above range is convex if the eigenvalues of  $N$  form an acute-angled or right-angled triangle inscribed to the unit circle.

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**Title** Semipositivity of matrices over the  $n$ -dimensional ice cream cone and some related questions

**Sachindranath Jayaraman** (sachindranathj@iisertvm.ac.in; sachindranathj@gmail.com), School of Mathematics, IISER Thiruvananthapuram, India.

**Co-authors** Chandrashekar Arumugasamy and Vatsalkumar N. Mer.

**Abstract:** An  $m \times n$  matrix  $A$  with real entries is said to be semipositive if there exists an  $x \geq 0$  such that  $Ax > 0$ , where the inequalities are understood componentwise. Our objective is to characterize semipositivity of matrices over the Lorentz or ice cream cone in  $\mathbb{R}^n$ , defined by  $\mathcal{L}_+^n = \{x = (x_1, \dots, x_n)^t \in \mathbb{R}^n \mid x_n \geq 0, \sum_{i=1}^{n-1} x_i^2 \leq x_n^2\}$ . We also investigate products of the form  $A_1 A_2^{-1}$ , where  $A_1$  is either positive or semipositive and  $A_2$  is positive and invertible. We hope to apply our results to linear complementarity problems.

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**Title:** An Algebraic Technique for Zeros of Unilateral Split Quaternion Polynomials

**Speaker:** Tongsong Jiang (jiangtongsong@sina.com), Heze University and Linyi University, China.

**Abstract:** This paper, by means of real representation of a split quaternion matrix, studies the solutions of split quaternion linear equation, unilateral quadratic equation with one unknown, and the linear equation with quadratic constraint. This paper also investigates the zeros of unilateral split quaternion polynomial of degree  $n$ , and gives an algebraic technique for finding the zeros of the unilateral split quaternion polynomial.

**Co-authors:** Zhaozhong Zhang, Ziwu Jiang.

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**Title:** New estimations of the upper bounds for the nuclear norm of a tensor

**Speaker:** Xu Kong (xu.kong@hotmail.com), Liaocheng University, Liaocheng, China.

**Abstract:** In this short talk, I will discuss the upper bounds for the nuclear norm of a tensor. New upper bounds for the nuclear norm of a tensor will be established by considering the orthogonal rank and the structure of a given tensor. Meanwhile, the relations between the nuclear norm of a tensor and the nuclear norm of its unfolding matrices will also be discussed. Finally, some conclusions will be given.

**Co-authors:** Jicheng Li and Xiaolong Wang.

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**Title:** Non-decomposable extremal positive bi-linear maps between two by two matrices

**Speaker:** Seung-Hyeok Kye (kye@snu.ac.kr), Seoul National University, Seoul, Korea.

**Abstract:** Positive bi-linear maps between matrix algebras play important roles to detect tri-partite entanglement by the duality between bi-linear maps and tri-tensor products. We construct a non-decomposable positive bi-linear map between  $2 \times 2$  matrices which generates an extreme ray in the cone of all positive bi-linear maps. In fact, it is exposed with respect to the duality mentioned above, and so detects entanglement with nonzero volume.

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**Title:** On the product of diagonal elements of  $3 \times 3$  normal matrices

**Speaker:** Pan-Shun Lau (panlau@connect.hku.hk), The Hong Kong Polytechnic University, Hong Kong.

**Co-author:** Nung-Sing Sze, The Hong Kong Polytechnic University, Hong Kong.

**Abstract:** Let  $A$  be a  $3 \times 3$  normal matrix. For  $1 \leq k \leq 3$ , let

$$W_k^\Pi(A) := \left\{ \prod_{i=1}^k d_i(U^*AU) : U \text{ is unitary} \right\},$$

where  $d_i(X)$  denotes the  $i$ -th diagonal element of  $X$ . In this talk, we will discuss the geometrical properties, such as convexity and star-shapedness, of  $W_k^\Pi(A)$ . In particular, we will show that  $W_2^\Pi(A)$  is star-shaped.

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**Title:** Apply proportional matrix congruence to classify real solvable algebras

**Speaker:** Anh-Vu Le (vula@uel.edu.vn) University of Economics and Law, VNU-HCMC, Vietnam

**Abstract:** Classifying all Lie algebras of dimension less than 4 is an elementary exercise. However, when considering Lie algebras of dimension  $n$  ( $n \geq 4$ ), complete classifications are much harder. As it has long been well known, there exist three different types of Lie algebras: the semi-simple, the solvable and those which are neither semi-simple nor solvable. By the Levi-Maltsev Theorem [13] in 1945, any finite-dimensional Lie algebra over a field of characteristic zero can be expressed as a semidirect sum of a semi-simple subalgebra and its maximal solvable ideal. It reduces the task of classifying all finite-dimensional Lie algebras to obtaining the classification of semi-simple and of solvable Lie algebras. The problem of the classification of semi-simple Lie algebras over the complex field has been completely classified by Killing, E. Cartan [4] in 1894, over the real field by F. R. Gantmakher [8] in 1939. Although several classifications of solvable Lie algebras of small dimension are known, but the problem of the complete classification of the (real or complex) solvable Lie algebras is still open up to now. There are two ways of proceeding in the classification of solvable Lie algebras: by dimension or by structure. It seems to be very difficult to proceed by dimension in the classification of Lie algebras of dimension greater than 6. However, it is possible to proceed by structure, i.e. to classify solvable Lie algebras with a specific given property. In this report, we use proportional matrix congruence to classify by structure, up to an isomorphism, the class consists of all solvable real Lie algebras whose first derived ideal have dimension one or two.

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**Title:** On spectra of matrix polynomials

**Speaker:** Cong Trinh Le (lecongtrinh@qnu.edu.vn), Quy Nhon University, Vietnam.

**Abstract:** In this talk we present some new results on the optimal matching distance between the spectra of two matrix polynomials. Moreover, we study the stability of the spectra of matrix polynomials and related topics.

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**Title:** Rhaly operators on small weighted Hardy spaces

**Speaker:** Hai-Khoi Le (lhkhai@ntu.edu.sg) Nanyang Technological University (NTU), Singapore

**Abstract:** Given a sequence of complex numbers  $\mathbf{a} = (a_n)_{n \geq 0}$ , the *Rhaly matrix* (a generalization of  $p$ -Cesàro matrices)  $R_{\mathbf{a}}$  is defined as the lower triangular matrix with constant row-segments

$$R_{\mathbf{a}} = \begin{bmatrix} a_0 & 0 & 0 & 0 & \dots \\ a_1 & a_1 & 0 & 0 & \dots \\ a_2 & a_2 & a_2 & 0 & \dots \\ a_3 & a_3 & a_3 & a_3 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

The action of Rhaly matrices on sequence spaces  $\ell^p$  and other sequence spaces is studied in many papers. However, there has been little research on Rhaly matrices acting on functional spaces. In this talk, we study several properties of Rhaly operators on small weighted Hardy spaces of holomorphic functions in the unit

disc  $D$ . In particular, we obtain criteria for boundedness, compactness and  $p$ -Schatten class membership of such operators

The results are based on joint works with P.L. Tan and M.L. Doan.

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**Title:** Low rank solutions to differentiable systems over matrices and applications

**Speaker:** Thanh Hieu Le (lethanhhieu@qnu.edu.vn), Quy Nhon University, Binh Dinh, Vietnam.

**Abstract:** A differentiable system in this talk means the one of equations that are described by differentiable real functions in real matrix variables. This talk discusses an algorithm for finding minimal rank solutions to such a system over (arbitrary and/or several structured) matrices by using the generalized Levenberg-Marquardt method (LM-method) for solving least squares problems. We then apply this algorithm to solve several problems in engineering such as the low-rank matrix completion problem, the low-dimensional Euclidean embedding one. Some numerical experiments illustrate the validity of the approach.

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**Title:** New random perturbation intervals of symmetric eigenvalue problem

**Speaker:** Hanyu Li (hyli@cqu.edu.cn;lihy.hy@gmail.com), Chongqing University, Chongqing, China.

**Co-author:** Chuanfu Xiao, Chongqing University.

**Abstract:** In this talk, we will first introduce a new random perturbation interval of symmetric eigenvalue problem. Under the random perturbation in the interval, the simple eigenvalue of the original matrix is still simple. Then, we introduce the corresponding result for the case of multiple eigenvalues. Finally, numerical experiments are presented to illustrate the new results.

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**Title:** 2-local Lie derivation and 2-local Lie automorphism on semi-finite factor von Neumann algebras

**Speaker:** Lei Liu (leiliu@mail.xidian.edu.cn), Xidian University, Xi'an, P.R. China.

**Abstract:** Let  $\mathcal{A}$  be an associative algebra (ring). A (not necessarily linear) map  $\delta : \mathcal{A} \rightarrow \mathcal{A}$  is called a 2-local Lie derivation(automorphism) if for each  $A$  and  $B$  in  $\mathcal{A}$ , there is a Lie derivation(automorphism)  $\delta_{A,B}$ , depending on  $A$  and  $B$ , such that  $\delta(A) = \delta_{A,B}(A)$  and  $\delta(B) = \delta_{A,B}(B)$ . In this talk, we describe the 2-local Lie derivation and 2-local Lie automorphism on semi-finite factor von Neumann algebras.

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**Title:** Merging of positive maps and its applications

**Speaker:** Marcin Marciniak (matmm@ug.edu.pl), University of Gdańsk, Gdańsk, Poland.

**Co-author:** Adam Rutkowski, University of Gdańsk, Gdańsk, Poland.

**Abstract:** In recent years positive maps on operator algebras began to play a significant role in various branches of mathematical physics. For instance, in quantum information theory they became an important tool for detecting entanglement while in the theory of dynamical system they serve as a natural generalization of dynamical maps. The talk deals with the problem of characterization of positive linear maps  $\phi : B(K) \rightarrow B(H)$  where  $K, H$  are finite dimensional Hilbert spaces. After pioneer work of Erling Størmer [5] (see also [6] for a review) several papers appeared with several examples (let [1, 2, 4] serve as representatives). However, in spite of great efforts of many mathematicians the classification of positive linear maps on  $C^*$ -algebras is still an open problem. Even in the finite dimensional case the situation is unclear. For example, no algebraic formula for general positive map between matrix algebras is known.

We present a general method for constructing wide class of positive maps. Assume that  $K_1, K_2, H_1, H_2$  are finite dimensional Hilbert spaces. Given two positive maps  $\phi_i : B(K_i) \rightarrow B(H_i)$ ,  $i = 1, 2$ , we construct a new linear map  $\phi : B(H) \rightarrow B(K)$ , where  $K = K_1 \oplus K_2 \oplus \mathbf{C}$ ,  $H = H_1 \oplus H_2 \oplus \mathbf{C}$ , by means of some additional ingredients such as operators and functionals. We call it a *merging* of maps  $\phi_1$  and  $\phi_2$ .

The aim of this talk is to present properties of this construction. In particular, we describe conditions for positivity of  $\phi$ , as well as for 2-positivity, complete positivity, optimality and indecomposability. In particular, we show that for a pair composed of 2-positive and 2-copositive maps, there is an indecomposable merging of them.

A particular case is the situation when  $\phi_1$  is completely positive while  $\phi_2$  is completely copositive. We show that there exists a *canonical merging*. It turns out that it is an exposed positive map when  $\phi_1$  and  $\phi_2$  are extremal ones. This result provides a wide class of new examples of exposed positive maps.

As an application, new examples of entangled PPT states are described. Moreover, we will present possible applications to a construction of bound entangled NPT states.

All above considerations will be illustrated by examples in the most simple case  $K_1 = K_2 = H_1 = H_2 = \mathbf{C}$ . The talk is based on the recent publication in LAA [3].

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**Title:** On sums of strictly  $k$ -zero matrices over arbitrary fields

**Speaker:** Little Hermie B. Monterde (lbmonterde@up.edu.ph), University of the Philippines Manila, Manila, Philippines.

**Co-author:** Agnes T. Paras, University of the Philippines Diliman, Quezon City, Philippines.

**Abstract:** Let  $k$  be an integer such that  $k \geq 2$ . An  $n$ -by- $n$  matrix  $A$  is said to be strictly  $k$ -zero if  $A^k = 0$  and  $A^m \neq 0$  for all positive integers  $m$  with  $m < k$ . Suppose  $A$  is an  $n$ -by- $n$  matrix over a field with at least three elements. We show that if  $A$  is a nonscalar matrix with zero trace, then i)  $A$  is a sum of four strictly  $k$ -zero matrices for all  $k \in \{2, \dots, n\}$ ; and ii)  $A$  is a sum of three strictly  $k$ -zero matrices for some  $k \in \{2, \dots, n\}$ . We prove that if  $A$  is a scalar matrix with zero trace, then  $A$  is a sum of five strictly  $k$ -zero matrices for all  $k \in \{2, \dots, n\}$ . We also determine the least positive integer  $m$  such that every square complex matrix  $A$  with zero trace is a sum of  $m$  strictly  $k$ -zero matrices for all  $k \in \{2, \dots, n\}$ .

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**Title:** Unitary similarity problem for determinantal representations

**Speaker** Hiroshi Nakazato (nakahr@hirosaki-u.ac.jp), Hirosaki University, Hirosaki, Japan.

**Co-author:** Mao-Ting Chien (Soochow University).

**Abstract.** Let  $A$  be an  $n$ -by- $n$  complex matrix. The Helton-Vinnikov theorem implies that there exists an  $n \times n$  complex symmetric matrix satisfying  $\det(x\Re(A) + y\Im(A) + zI_n) = \det(x\Re(S) + y\Im(S) + zI_n)$ . They provides an explicit method to construct  $S$  from  $A$ . Concerning this result, we treat two questions. The first: What type of matrices are unitarily similar to symmetric matrices? We show that Toeplitz matrices and unitary bordering matrices satisfy such a condition. The second: What matrix  $A$  is unitarily similar to the Helton-Vinnikov symmetric matrix? We show that some unitary bordering matrix does not satisfy this condition.

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**Title:** Quantum Gauss-Jordan Elimination and simulation of accounting principles on quantum computers

**Speaker:** Van-Minh Nguyen (nvminh07@gmail.com), Thuong Tin High School, Hanoi, Vietnam.

**Abstract:** The paper is devoted to a version of Quantum Gauss-Jordan Elimination and its applications. In the rst part, we construct the Quantum Gauss-Jordan Elimination (QGJE) Algorithm and estimate the complexity of computation of Reduced Row Echelon Form (RREF) of  $N \times N$  matrices. The main result asserts that QGJE has computation time is of order  $2N/2$ . The second part is devoted to a new idea of simulation of accounting by quantum computing. We rst expose the actual accounting principles in a pure

mathematics language. Then, we simulate the accounting principles on quantum computers. We show that, all accounting actions are exhausted by the described basic actions. The main problems of accounting are reduced to some system of linear equations in the economic model of Leontief. In this simulation, we use our constructed Quantum Gauss-Jordan Elimination to solve the problems and the complexity of quantum computing is a square root order faster than the complexity in classical computing.

**Title:** Unitary equivalent classes of one-dimensional quantum walks

**Speaker:** Hiromichi Ohno (h\_ohno@shinshu-u.ac.jp), Shinshu University, Nagano, Japan

**Abstract:** This study investigates unitary equivalent classes of one-dimensional quantum walks. A quantum walk is defined by a pair  $(U, \{\mathcal{H}_v\}_{v \in V})$ , where  $V$  is a countable set,  $\{\mathcal{H}_v\}_{v \in V}$  is a family of separable Hilbert spaces, and  $U$  is a unitary operator on  $\mathcal{H} = \bigoplus_{v \in V} \mathcal{H}_v$ . In this talk, we consider one-dimensional (two-state) quantum walks, i.e.,  $V = \mathbb{Z}$  and  $\mathcal{H}_v = \mathbb{C}^2$ , and we determine the unitary equivalent classes of several types of one-dimensional quantum walks.

**Title:** Łojasiewicz inequalities with explicit exponent for smallest singular value functions

**Speaker:** Tien-Son Phạm (sonpt@dlu.edu.vn), University of Dalat, Dalat, Vietnam.

**Co-author:** Sĩ-Tiếp Đinh (dstiep@math.ac.vn), Institute of Mathematics, VAST, Hanoi, Vietnam

**Abstract:** Let  $F(x) := (f_{ij}(x))_{i=1, \dots, p; j=1, \dots, q}$ , be a  $(p \times q)$ -real polynomial matrix and let  $f(x)$  be the smallest singular value function of  $F(x)$ . In this paper, we first give the following nonsmooth version of Łojasiewicz gradient inequality for the function  $f$  with an explicit exponent: For any  $\bar{x} \in \mathbb{R}^n$ , there exist  $c > 0$  and  $\epsilon > 0$  such that we have for all  $\|x - \bar{x}\| < \epsilon$ ,

$$\inf\{\|w\| : w \in \partial f(x)\} \geq c|f(x) - f(\bar{x})|^{1-\tau},$$

where  $\partial f(x)$  is the limiting subdifferential of  $f$  at  $x$ ,  $d := \max_{i=1, \dots, p; j=1, \dots, q} \deg f_{ij}$ ,  $R(n, d) := d(3d - 3)^{n-1}$  if  $d \geq 2$  and  $R(n, d) := 1$  if  $d = 1$ , and  $\tau := \frac{1}{R(n+p, 2d+2)}$ . Then we establish some versions of Łojasiewicz inequality for the distance function with explicit exponents, locally and globally, for the smallest singular value function  $f(x)$  of the matrix  $F(x)$ .

This is a joint work with Sĩ-Tiếp Đinh.

**Title:** Using compound and companion matrices to obtain improved bounds for zeros of polynomials.

**Speaker:** Rajesh Pereira (pereirar@uoguelph.ca), University of Guelph, Guelph ON, Canada.

**Co-author:** Mohammad Ali Vali, Shahid Bahonar University of Kerman

**Abstract:** We use compound matrices to give a new generalization of the classical Cauchy and Fujiwara bounds on the zeros of polynomials. We will look at further applications of compound matrix techniques to study other matrices useful in the analytic theory of polynomials. It turns out that in these cases the compound matrices may be sparse even if the original matrices are not.

**Title:** Nonexistence of entire positive solution to cooperative parabolic systems

**Speaker:** Quoc-Hung Phan (hungpqmath@gmail.com), Duy Tan University, Vietnam.

**Abstract:** We prove the nonexistence of entire positive solutions to cooperative parabolic systems. By nontrivial modifications of the techniques of Gidas and Spruck and of Bidaut-Vron, we partially improve the results of Quittner in space dimensions  $N > 2$ . In particular, our result solves the important case of the parabolic Gross-Pitaevskii system in space dimension  $N = 3$ .

**Co-authors:** Philippe Souplet.

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**Title:** Strong  $k$ -commutativity preservers on complex standard operator algebras

**Speaker:** Xiaofei Qi (qixf1980@126.com), Shanxi University, Taiyuan, China.

**Co-authors:** Jinchuan Hou, Taiyuan University of Technology, Taiyuan, China.

**Abstract:** Let  $X$  be a complex Banach space with  $\dim X \geq 2$  and  $\mathcal{A}$  be a standard operator algebra on  $X$ . Let  $k$  be a positive integer. For  $A, B \in \mathcal{A}$ , the  $k$ -commutator of  $A$  and  $B$  is defined by  $[A, B]_k = [[A, B]_{k-1}, B]$  with  $[A, B]_0 = A, [A, B]_1 = [A, B] = AB - BA$ . Let  $\Phi : \mathcal{A} \rightarrow \mathcal{A}$  be a map. We say that  $\Phi$  is strong  $k$ -commutativity preserving if  $[\Phi(A), \Phi(B)]_k = [A, B]_k$  for any  $A, B \in \mathcal{A}$ . In this paper, it is shown that, if the range of  $\Phi$  contains all operators of rank  $\leq 1$ , then  $\Phi$  is strong  $k$ -commutativity preserving if and only if there exist a functional  $h : \mathcal{A} \rightarrow \mathbb{C}$  and a scalar  $\lambda \in \mathbb{C}$  with  $\lambda^{k+1} = 1$  such that  $\Phi(A) = \lambda A + h(A)I$  for all  $A \in \mathcal{A}$ .

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**Title:** Symmetry classes of tensors and semi-direct product of finite abelian groups

**Speaker:** Kijti Rodtes (kijt@nu.ac.th), Naresuan University, Thailand.

**Abstract:** "In the study of symmetry classes of tensors, finding examples of symmetry classes of tensors that possess an  $o^*$ -basis is of considerable interest. There are only few classes of groups that have been provided a necessary and sufficient condition for having such a basis. There is no general criterion for any finite groups yet. In this talk, we will provide a necessary and sufficient condition for the existence of  $o^*$ -basis of symmetry classes of tensors associated with semi-direct product of some finite abelian groups and, consequently, their wreath product."

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**Title:** An approach to immanant inequalities and their limiting behavior

**Speaker:** Ryo Tabata (tabata@ariake-nct.ac.jp), National Institute of Technology, Ariake College, Omuta (Fukuoka), Japan.

**Abstract:** An immanant is a special case of the generalized matrix functions labeled by Young diagrams. Schur's inequality (1917) and Lieb's permanent dominance conjecture (1966) assert respectively that the determinant is the least and the permanent is the largest of immanants. In this talk, we analyze the correlation matrix  $Y_n = (n/(n-1)\delta_{ij} - 1/(n-1))$ , which conjecturally gives sharper bounds of these inequalities, where  $\delta_{ij}$  is the Kronecker delta function. Motivated by Frenzen-Fischer's result (1993), i.e.  $\lim_{n \rightarrow \infty} \text{per } Y_n = e/2$ , we explore the behavior of immanants through the limit shape of Young diagrams. The Littlewood-Richardson rule, an important property to describe the representation of the symmetric group, will be applied as the key lemma.

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**Title:** Numerical ranges and semisimple Lie algebras

**Speaker:** Tin-Yau Tam (tamtiny@auburn.edu), Department of Mathematics and Statistics, Auburn University, AL 36849, USA

**Abstract:** Given an  $n \times n$  matrix  $A$ , the celebrated Toeplitz-Hausdorff theorem asserts that the classical numerical range  $\{x^*Ax : x \in \mathbb{C}^n : x^*x = 1\}$  is a convex set, where  $\mathbb{C}^n$  is the vector space of complex  $n$ -tuples and  $x^*$  is the complex conjugate transpose of  $x \in \mathbb{C}^n$ . Among many interesting generalizations, we will focus our discussion on those in the context of Lie structure, more precisely, compact connected Lie groups and semisimple Lie algebras. Some results and questions on convexity and star-shapedness will be presented.

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**Title:** A generalization of the Craig-Sakamoto theorem to Euclidean Jordan algebras

**Speaker:** Jiyuan Tao (jt@loyola.edu), Loyola University Maryland, Baltimore, USA.

**Co-authors:** Guoqiang Wang (Shanghai University of Engineering Science).

**Abstract:** Letac and Massam extended the Craig-Sakamoto theorem to Euclidean Jordan algebras. In this talk, we present another proof of this generalization by reformulating the result in terms of rank and determinant equalities and by proving the result in each of the simple Euclidean Jordan algebras.

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**Title:** Sums of  $H$ -Unitary Matrices

**Speaker:** Terrence Teh (tete@math.upd.edu.ph), University of the Philippines Diliman, Quezon City, Philippines.

**Co-authors:** Agnes T. Paras (University of the Philippines Diliman), Dennis I. Merino (Southeastern Louisiana University)

**Abstract:** Let  $H \in M_n$  be an nonsingular and Hermitian. A matrix  $A$  is said to be  $H$ -unitary if  $A^*HA = H$ . The set of  $H$ -unitary matrices forms a multiplicative group. However, the sum of  $H$ -unitary matrices need not be  $H$ -unitary. We discuss some previous results and show analogous or new properties for sums of  $H$ -unitary matrices. For example, we show that every matrix can be expressed as a sum of  $H$ -unitary matrices. We also characterize all matrices expressible as a sum of two  $H$ -unitary matrices.

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**Title:** The classification of cut edges due to the change in multiplicity of an eigenvalue.

**Speaker:** Kenji Toyonaga (toyonaga@kct.ac.jp), National Institute of Technology, Kitakyushu College, Japan.

**Abstract:** Given a real symmetric matrix whose graph has a cut edge, the cut edge can be classified in four categories, based upon the change in multiplicity of a particular eigenvalue, when the edge is removed. We show a necessary and sufficient condition for each possible classification of a cut edge. A special relationship is observed among 2-Parter edges, Parter edges and singly Parter vertices, when the graph is a tree. Then, we investigate the change in multiplicity of an eigenvalue based upon a change in an edge value. We show how the multiplicity of the eigenvalue changes depending upon the status of the edge and the edge value.

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**Title:** On modified Hermitian and skew-Hermitian iteration method for the space fractional coupled nonlinear Schrödinger equations

**Speaker:** Zeng-Qi Wang(wangzengqi@sjtu.edu.cn), Shanghai Jiaotong University, Shanghai, China.

**Co-authors:** Jun-Feng Yin and Quan-Yu Dou, Tongji University, Shanghai, China.

**Abstract:** The space fractional coupled nonlinear Schrodinger equations is a type of very important model in quantity physics. After discretized by an implicit conservative difference scheme, which is unconditionally stable, a complex linear equations is obtained. The imaginary part of the coefficient matrix is the identity matrix while the real part of it is a symmetric diagonal-plus-Toeplitz matrix. In this talk, we will introduce a matrix splitting iteration method to solve the complex symmetric linear systems based on the preconditioned modified Hermitian and skew-Hermitian splitting of the coefficient matrix, so that we name it the preconditioned modified Hermitian and skew-Hermitian splitting iteration method, or PMHSS iteration method in short. The iteration method has the following features: It has only real float arithmetics; The structure of the Toeplitz matrix is taken into account, thereby, the fast solver (FFT) is utilized in the iterations. Due to the eigenvalues distribution, we prove the unconditional convergence of the achieved iteration method. Moreover, we obtain the optimal parameter and the corresponding optimal spectral radius. Numerical behavior show that the method finds the satisfying solution in a few iterative steps, the iteration counter is independent with the differential order  $\alpha$ , the discretizing mesh size and even the initial condition of the Schrodinger equations.

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**Title:** On operator  $(r, s)$ -convex functions

**Speaker:** Khue TB Vo, University of Finance - Marketing, Ho Chi Minh City, Vietnam.

**Co-authors:** Thanh-Duc Dinh, Quy Nhon University, Vietnam and Trung-Hoa Dinh, University of North Florida

**Abstract:** Let  $r, s$  be positive numbers and  $J \subset \mathbb{R}^+$ . We define a new class of operator  $(r, s)$ -convex functions by the following inequality

$$f((\alpha A^r + (1 - \alpha)B^r)^{1/r}) \leq (\alpha f(A)^s + (1 - \alpha)f(B)^s)^{1/s},$$

where  $A, B$  are positive definite matrices and for any  $\alpha \in [0, 1]$ . We prove the Jensen, Hansen-Pedersen and Rado type inequalities for operator  $(r, s)$ -convex functions. Some equivalent conditions for a function  $f$  to become operator  $(r, s)$ -convex are provided.

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**Title:** The investigations on some Sylvester-type matrix equations

**Speaker:** Qing-Wen Wang (wqw@t.shu.edu.cn), Shanghai University, P.R. China

**Co-author:** Zhuoheng He, Auburn University, Auburn, USA.

**Abstract:** In this talk, we first review some results related Sylvester-type matrix equations and matrix decompositions. Then we investigate and analyze in detail the structure and properties of a simultaneous decomposition for fifteen matrices:  $A_i \in \mathbb{C}^{p_i \times t_i}$ ,  $B_i \in \mathbb{C}^{s_i \times q_i}$ ,  $C_i \in \mathbb{C}^{p_i \times t_{i+1}}$ ,  $D_i \in \mathbb{C}^{s_{i+1} \times q_i}$ , and  $E_i \in \mathbb{C}^{p_i \times q_i}$ , ( $i = 1, 2, 3$ ). We show that from this simultaneous decomposition we can derive some necessary and sufficient conditions for the existence of a solution to the system of two-sided coupled generalized Sylvester matrix equations with four unknowns  $A_i X_i B_i + C_i X_{i+1} D_i = E_i$ , ( $i = 1, 2, 3$ ). Apart from proving an expression for the general solutions to this system, we derive the range of ranks of these solutions using the ranks of the given matrices  $A_i, B_i, C_i, D_i$ , and  $E_i$ . We provide some numerical examples to illustrate our results. Moreover, we present a similar approach to consider the simultaneous decomposition for  $5k$  matrices and the system of  $k$  two-sided coupled generalized Sylvester matrix equations with  $k + 1$  unknowns  $A_i X_i B_i + C_i X_{i+1} D_i = E_i$ , ( $i = 1, \dots, k, k \geq 4$ ). The main results are also valid over the real number field and an arbitrary division ring. The findings of this paper extend some known results in the literature.

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**Title:** Several Questions about Tensors

**Speaker:** Fuzhen Zhang (zhang@nova.edu), Nova Southeastern University, Florida, USA

**Abstract:** This talk will present several research problems about tensors (multidimensional arrays).

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**Title:** On The spectral Turan-Type Results of graphs

**Speaker:** Xiao-Dong Zhang (xiaodong@sjtu.edu.cn), Shanghai Jiao Tong University, Shanghai, P.R. China.

**Co-author:** Ya-Lei Jin (zzuedujinyalei@163.com), Shanghai Normal University, Shanghai, P.R. China.

**Abstract:** In 1941, Turan proved the famous Turan theorem, i.e., If  $G$  is a graph which does not contain  $K_{r+1}$  as its subgraph, then the edge number of  $G$  is no more than  $T_{n,r}$ , Turan graph, which started the extremal theory of graphs. In this talk, we will introduce the spectral Turan-Type results which are incident with the adjacent matrix, signless Laplacian matrix. Moreover, some open problems in this field are proposed.

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